A Vertex-centered Discontinuous Galerkin Method for Fluid Flows

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Background and Motivation

There is a need in aeronautical industry to go from low-order finite volume methods (FVM) to high-order discontinuous Galerkin methods (DGM).

Current aeronautical industry, FVM:

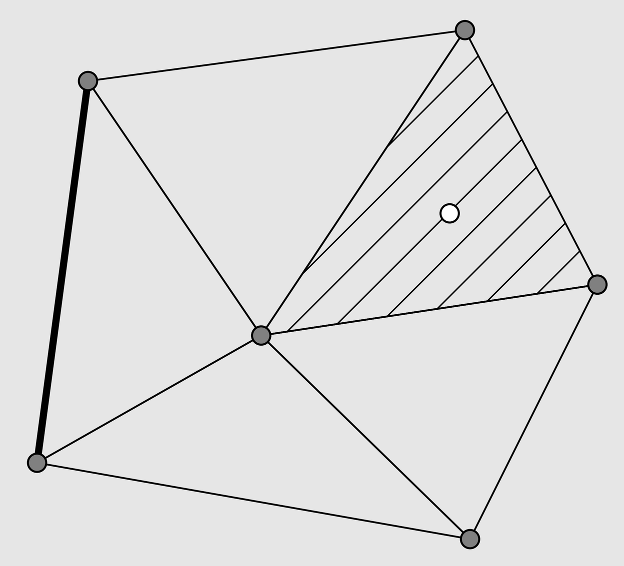
- Vertex-centered FVM
- Low-order
- Highy efficient and complex codes

Current state of the art, DGM:

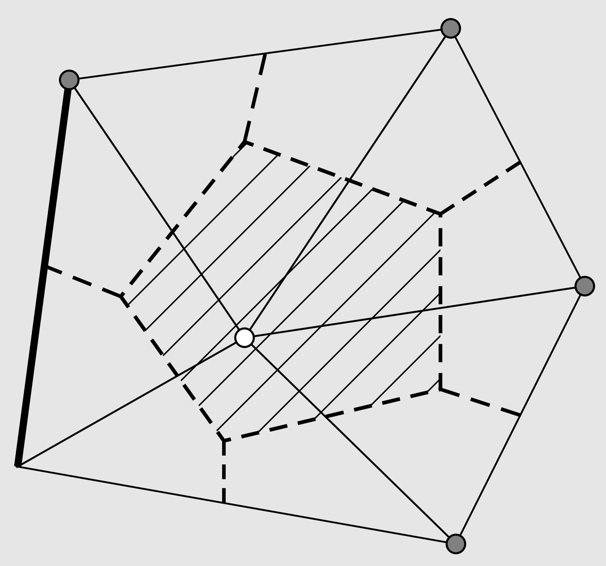
- Cell-centered DGM
- High-order
- No industrial codes, incompatible with existing codes

Our version of DGM:

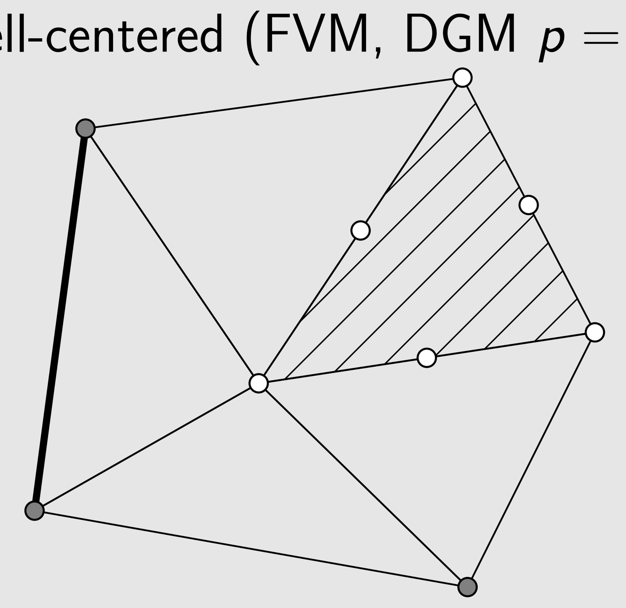
- Vertex-centered DGM
- High-order
- Low cost incorporation in existing industrial code, **Edge**



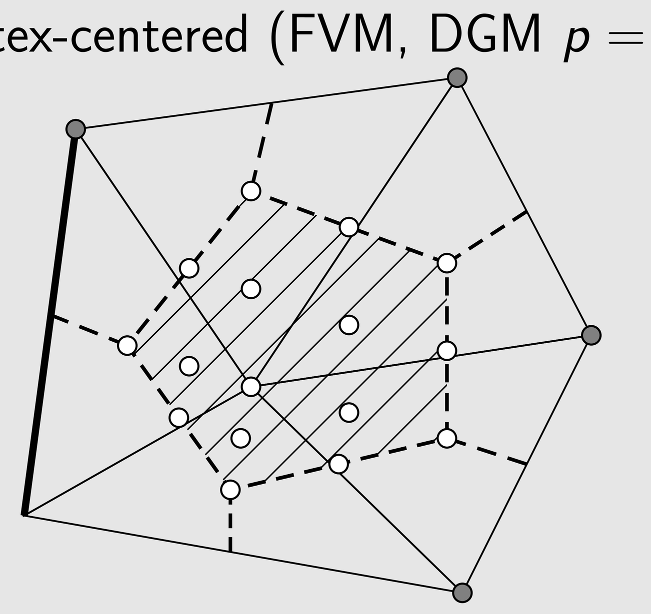
Cell-centered (FVM, DGM $p = 0$)



Vertex-centered (FVM, DGM $p = 0$)



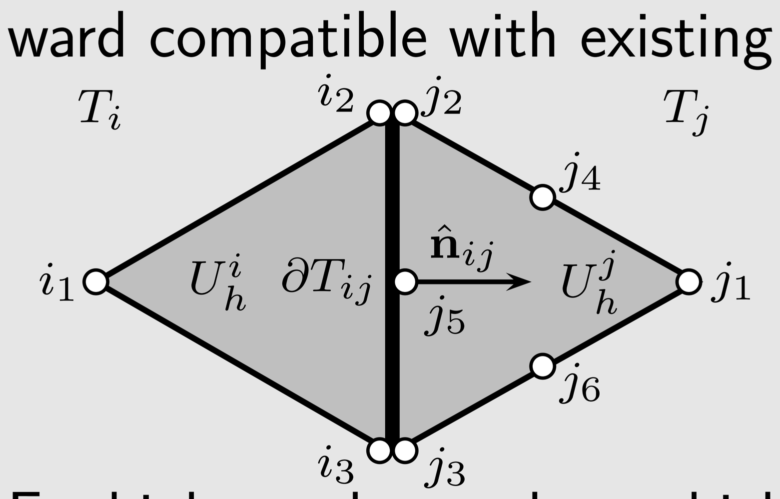
Cell-centered (DGM $p = 2$)



Vertex-centered (DGM $p = 2$)

Data Structures

More complex data structures are needed, but can be made backward compatible with existing structures in FVM codes.



FVM:

- $i_1, j_1, \mathbf{n}_{ij}$

DGM:

- i_1, i_2, i_3
- $j_1, j_2, j_3, j_4, j_5, j_6$
- $\mathbf{n}_{ij}, \mathbf{x}_{i1}, \mathbf{x}_{j1}, \mathbf{x}_{i2}, \mathbf{x}_{i3}$

For higher order meshes, which is needed for DGM, new data structures are also required to treat boundaries correctly without loss of accuracy. Variable orders in different elements is supported, as seen above with a $p = 1$ and $p = 2$ element.

Some Features Implemented

- Transport, Burger's, Euler 2$\mathbb{D}$
- Standard Lagrange elements
- *hp*-adaption
- High-order mesh
- Shock detection and capturing
- *hp*-Multigrid
- Local timestepping
- Visualization

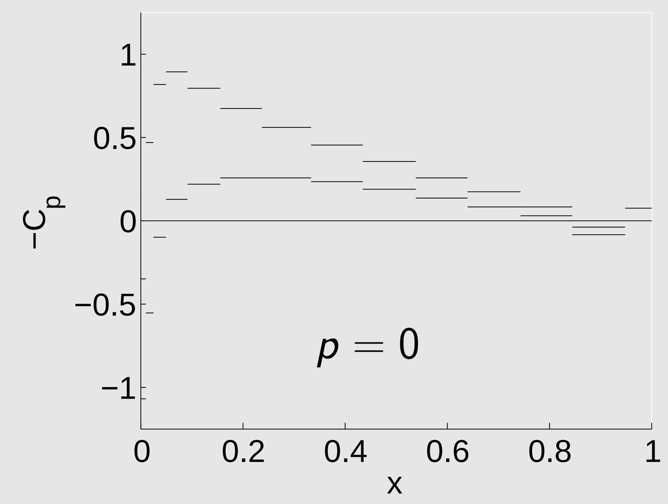
Results

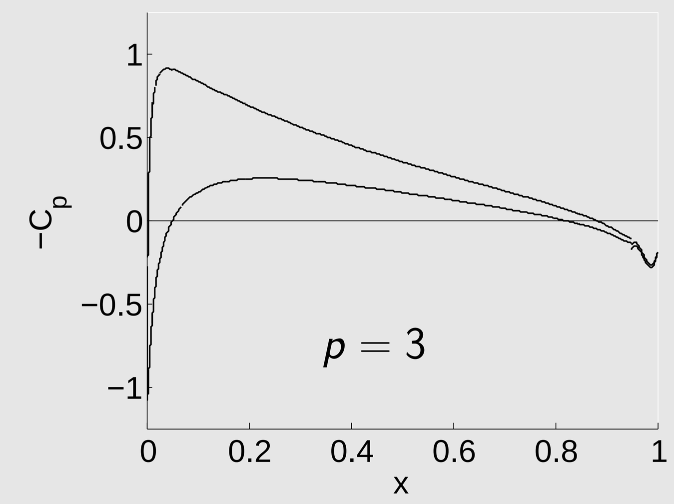
The $L^2(\Omega)$-error, $\|u - u_h\|_{L^2(\Omega)}$, numerical order of convergence, s, and the number of degrees of freedom N_{dof}, for the linear model problem, using macro elements based on Lagrange triangles of different orders p.

	constant, $p = 0$			linear, $p = 1$		
$h_{\max}$	$\|u-u_h\|_{L^2(\Omega)}$	s	N_{dof}	$\|u-u_h\|_{L^2(\Omega)}$	s	N_{dof}
h_0	1.8558×10^{-1}	–	185	3.5491×10^{-3}	–	1249
$2^{-1}h_0$	1.1685×10^{-1}	0.67	697	8.5294×10^{-4}	2.06	4793
$2^{-2}h_0$	6.8506×10^{-2}	0.77	2705	2.0608×10^{-4}	2.05	18769
$2^{-3}h_0$	3.8030×10^{-2}	0.85	10657	5.0122×10^{-5}	2.04	74273
$2^{-4}h_0$	2.0356×10^{-2}	0.90	42305	1.2322×10^{-5}	2.02	295489
$2^{-5}h_0$	1.0660×10^{-2}	0.93	168577	3.0512×10^{-6}	2.01	1178753

	quadratic, $p = 2$			cubic, $p = 3$		
$h_{\max}$	$\|u-u_h\|_{L^2(\Omega)}$	s	N_{dof}	$\|u-u_h\|_{L^2(\Omega)}$	s	N_{dof}
h_0	2.9501×10^{-4}	–	3337	5.8787×10^{-6}	–	6449
$2^{-1}h_0$	4.3033×10^{-5}	2.78	12905	3.3308×10^{-7}	4.14	25033
$2^{-2}h_0$	5.8880×10^{-6}	2.87	50737	1.9522×10^{-8}	4.09	98609
$2^{-3}h_0$	7.7203×10^{-7}	2.93	201185	1.1674×10^{-9}	4.06	391393
$2^{-4}h_0$	1.0202×10^{-7}	2.92	801217	7.0708×10^{-11}	4.05	1559489
$2^{-5}h_0$	1.3564×10^{-8}	2.91	3197825	4.3273×10^{-12}	4.03	6225793

Below is shown the pressure coefficient C_p of NACA0012, $M_\infty = 0.5, \alpha = 2^\circ$, with improved results for $p = 3$ and fewer element compared to standard FVM.





References

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III S.-E. Ekström and M. Berggren. Incorporating a discontinuous Galerkin method into the existing vertex-centered edge-based finite volume solver Edge. In ADIGMA - A European Initiative on the Development of Adaptive Higher-Order Variational Methods for Aerospace Applications, volume 113 of *Notes on Numerical Fluid Mechanics and Multidisciplinary Design*, pp 39-52, Springer-Verlag, Berlin, 2010.

Conclusion

The vertex-centered discontinus Galerkin method is a cost-effective and viable extension to higher orders of existing industrial low-order finite volume method codes.

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