Part II: The Road Map of Machine Learning Algorithms

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Introduction

- Kernel methods: Support vector machine
- Ensemble methods:
    - Random forest
    - Adaboost
    - Artificial Neural Network

Kernel methods

- Feature mapping: $\phi : \mathbb{R}^2 \rightarrow \mathbb{R}^3$

$$(x_1, x_2) \rightarrow (x_1^2, \sqrt{2}x_1x_2, x_2^2)$$

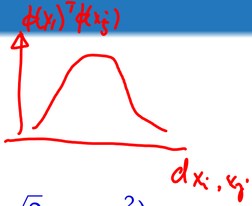
- Inexplicit mapping: ϕ

$x_i^T x_j$

$\phi(x)^T \phi(x)$

$$(x_1^2, \sqrt{2}x_1x_2, x_2^2)\begin{pmatrix} x_1^2 \\ \sqrt{2}x_1x_2 \\ x_2^2 \end{pmatrix} = \underline{(x_1^2 + x_2^2)}^2$$

Kernel methods



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- Kernel function: $\underline{\kappa_\phi(\mathbf{x}_i, \mathbf{x}_j)} = \cancel{\phi}(\mathbf{x}_i)^T\cancel{\phi}(\mathbf{x}_j)$
- Polynomial kenerl and Gaussian Kernel (RBF).

$$K_\phi(\mathbf{x}_i, \mathbf{x}_j) = (c + \gamma\, \mathbf{x}_i^T \mathbf{x}_j)^M \qquad K_\phi(\mathbf{x}_i, \mathbf{x}_j) = \exp\left\{-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{\sigma^2}\right\}$$

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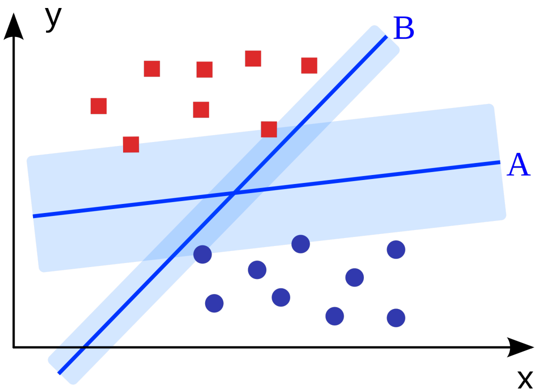
- Polynomial kenerl and Gaussian Kernel (RBF).
- Kernel tricks: obtain a non-linear model by embedding kernel function into different linear algorithms.
- Kernel PCA, Kernel regression, Support vector machine.

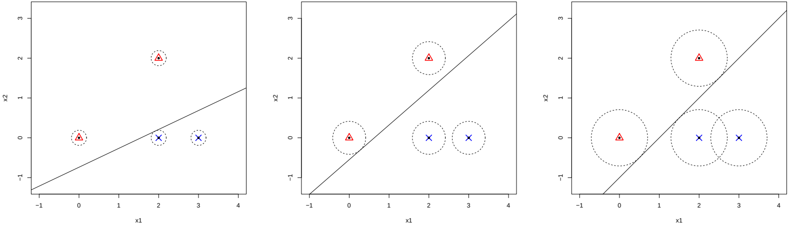
Maximum Margin Classifier

- The margin of the decision boundary should be as large as possible.



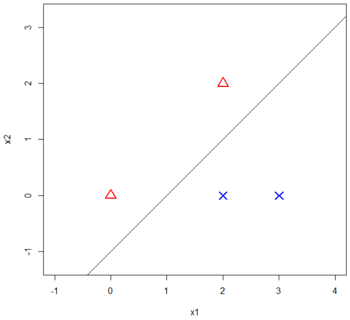
Maximum Margin Classifier

- A statistical illustration of this idea:



$\varepsilon \sim \mathcal{N}(0, \sigma_A^2)$ $\quad \sigma_B^2 \quad \sigma_C^2$

Support Vectors



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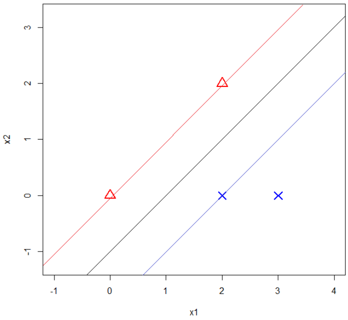

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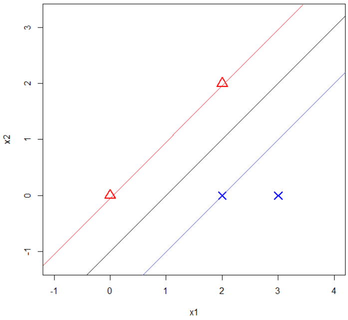

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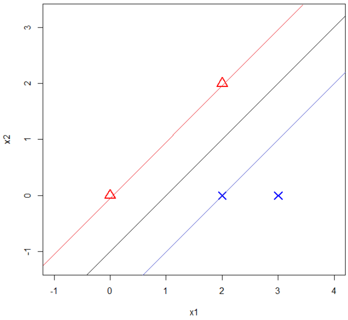

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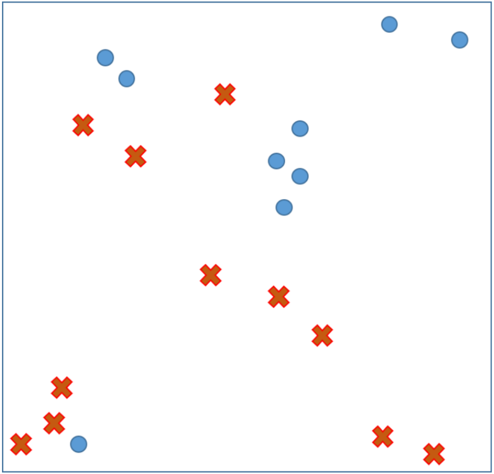
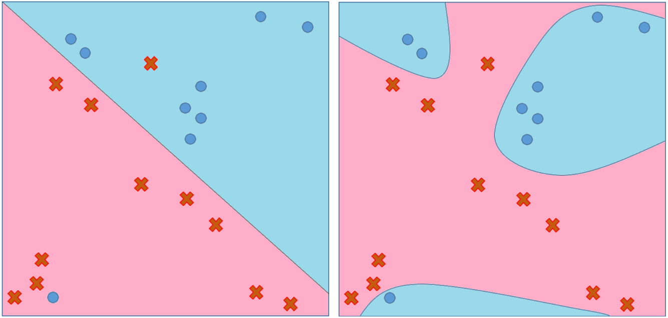
- Kernelized MMC $\rightarrow$ support vector machine

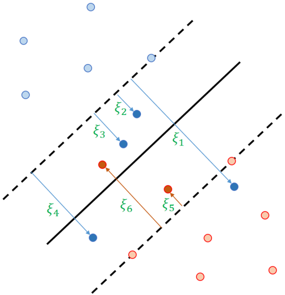


Over fitting in SVM



Soft Margin Classifier

- IDEA: We give up some high noise cases.
- For each observation, we introduce a fuzzy (Slackness) parameter, $\xi_i \geq 0$.
- Hyper-parameter: large C, less noise tolerance, high cost.



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- Tuning: exponential growing sequesces of C and σ, e.g. $2^{-5}, 2^{-3}, ..., 2^{15}$.

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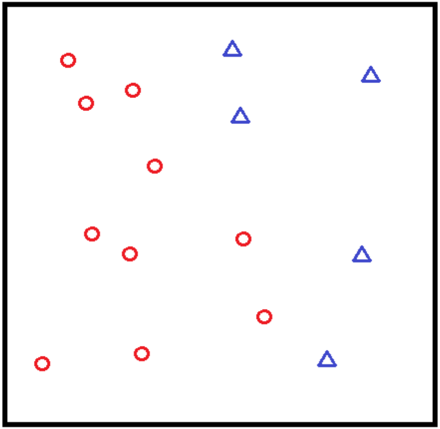
- Non-uniform Aggregation. Adaboost.

Uniform Aggregation



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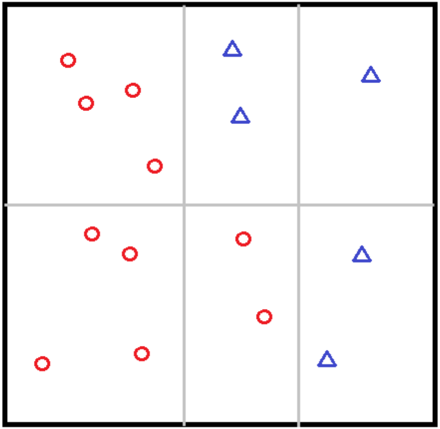

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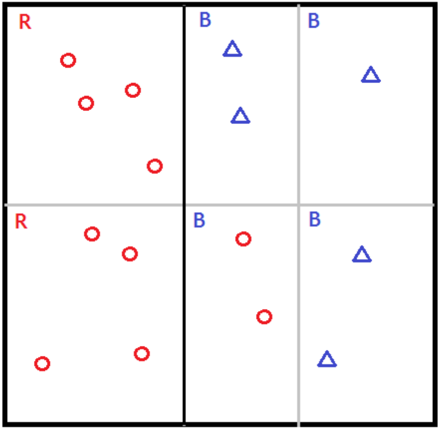

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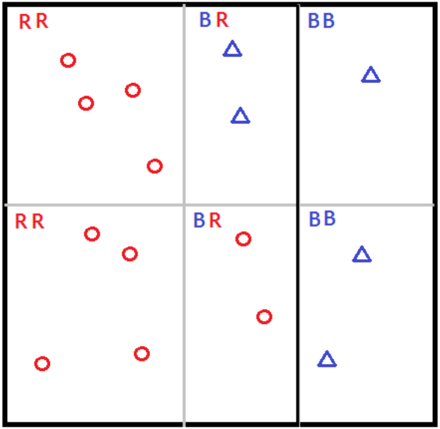

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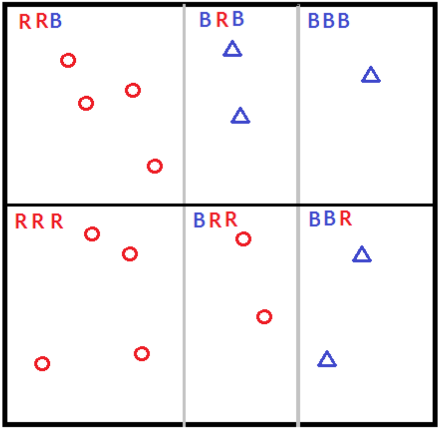

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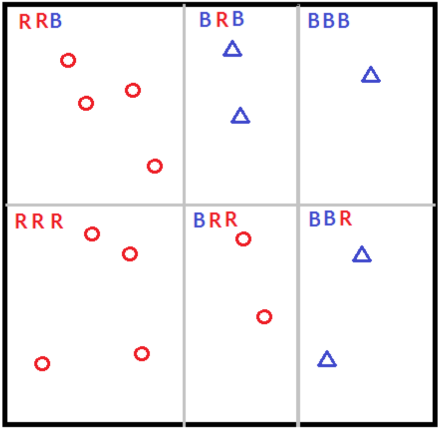

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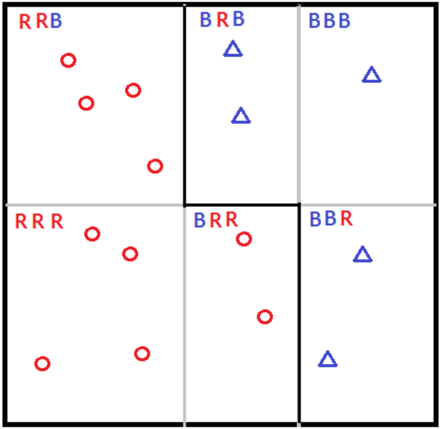

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$Var(X) = E(X - E(X))^2$

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Decision Tree

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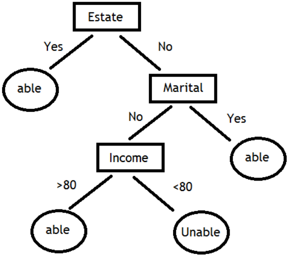
2	0	0	M	100
3	0	0	S	70
4	0	1	M	120
5	1	0	D	95
6	0	0	M	60
7	0	1	D	220
8	1	0	S	85
9	0	0	M	75
10	1	0	S	90



Decision Tree

Key: Sequentially split the feature space into small pieces.

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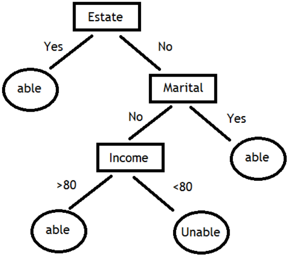

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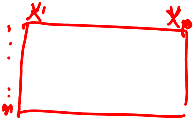

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4. Apply uniform aggregation method on those trees, then we have our random forest model.

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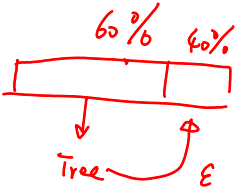

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- Self-validation.

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- Replace the feature x_i by permuted x_i^P
- Then we can measure the importance of x_i by the difference of performance

$$\mathcal{I}(x_j) = Err_{OOB}(x_j) - Err_{OOB}(x_j^p)$$

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  - Choose the best forest given the OOB errors.
  - Fix N_t and choose different numbers around the squar root of the total number of feature variables.
  - Build up the classifier, then select the best given OOB errors.

Boosting and Adaboosting

- Uniform aggregation: Bagging, Random Forest.

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Adaboost

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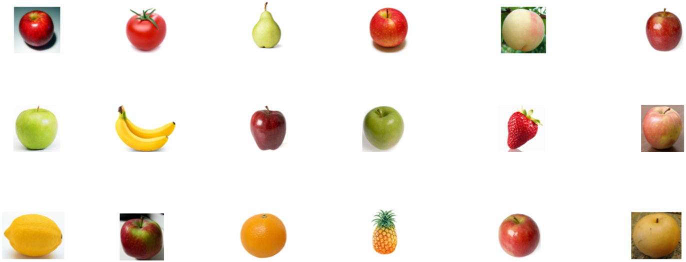

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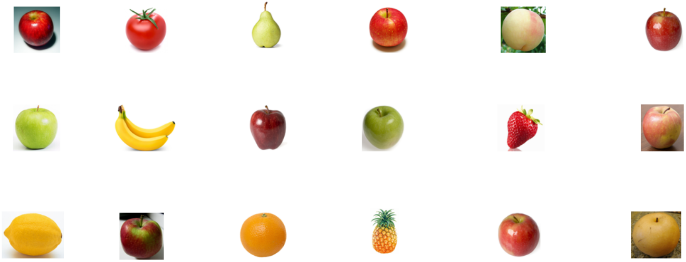
Circle



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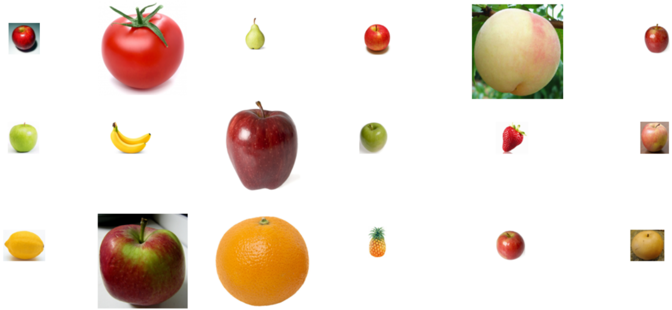

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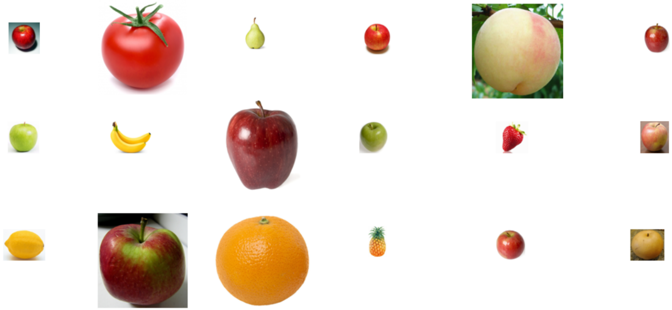
Circle $+$ **Red**



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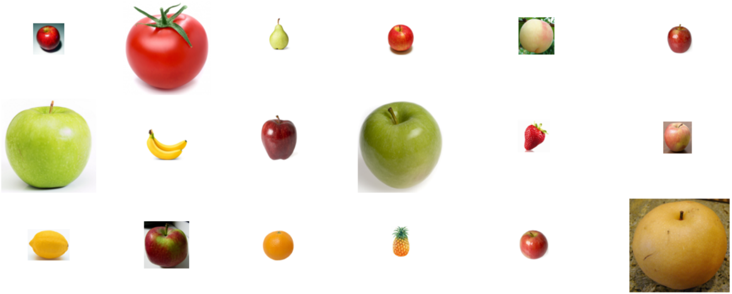
Circle + **Red**



Adaboost

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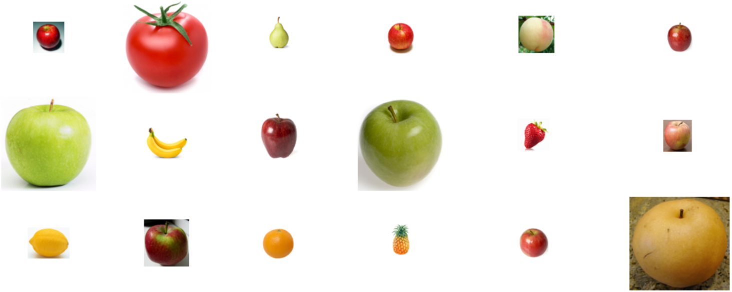
Circle + **Red** + **Green**



Adaboost

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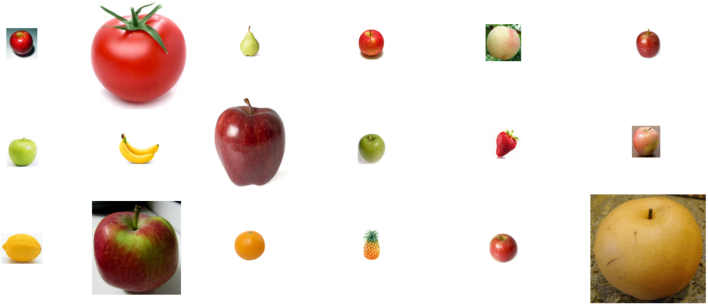
Circle + Red + Green



Adaboost

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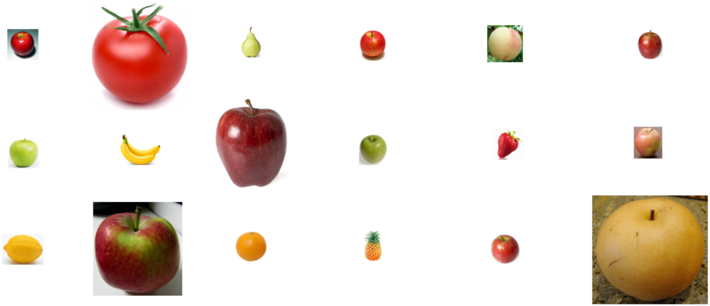
Circle + **Red** + **Green** + **Stem**



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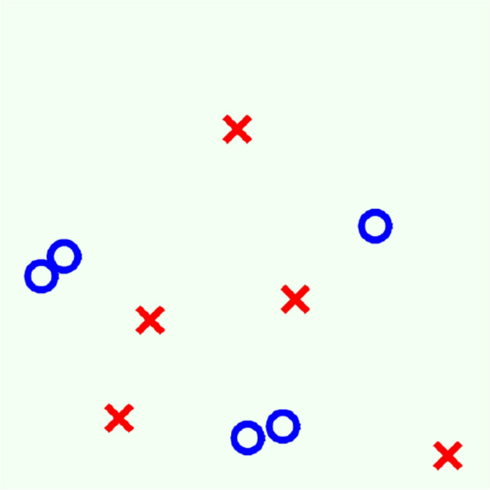
- Adaboost: a reweight scheme, such that the mis-classified examples get more weights in the next round.

Toy example in Action



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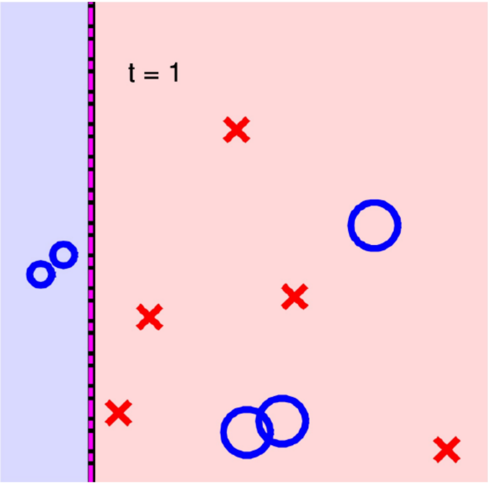

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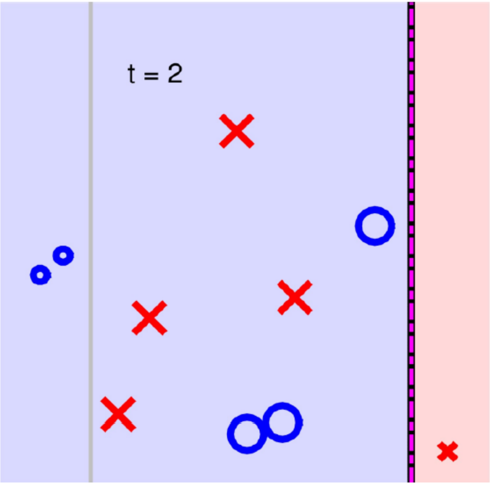

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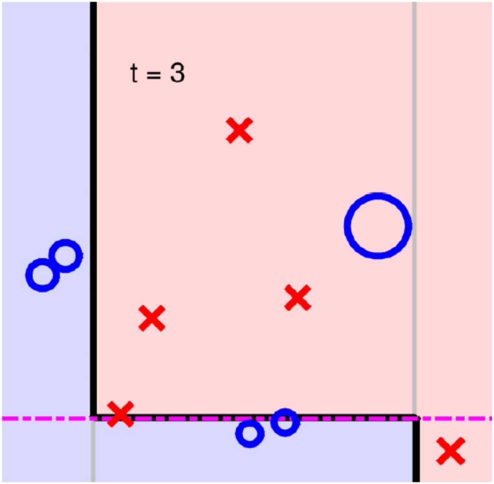

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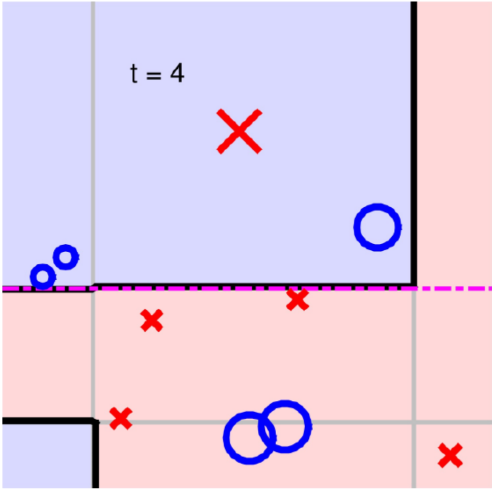

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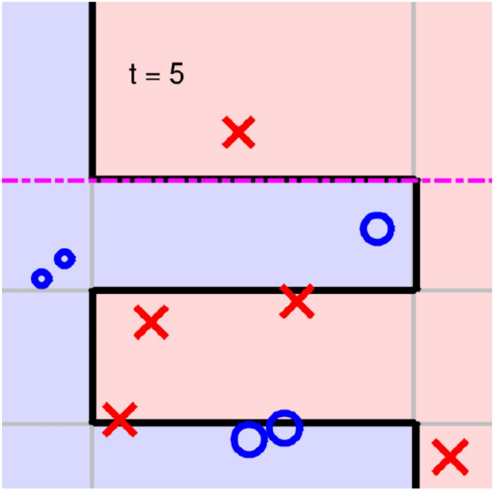

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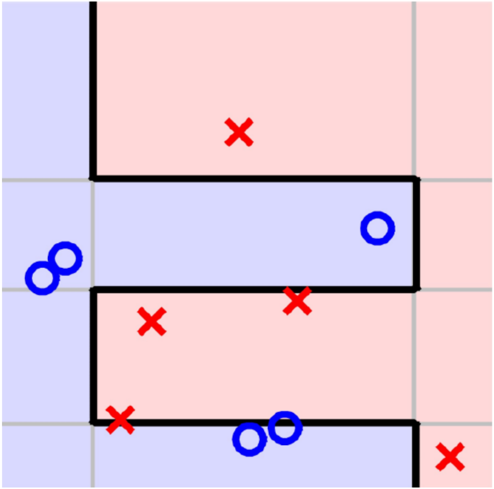

ANN in Ensemble methods

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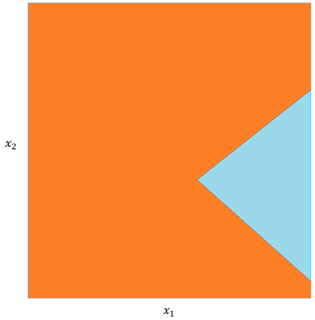
- Random forest = Bagging + Decision tree
- **Question**: Can we learn f_j as a perceptron, $f_j = Sign\left(\mathbf{w}^T\mathbf{x} + w_0\right)$?
- **Question**: Can we learn different weights for different perceptrons from the target variable $\mathbf{y}$, such that the final model $F(\mathbf{x})$, is a non-uniform aggregation of those perceptrons?

$$F(\mathbf{x}) = Sign\left(\alpha_0 + \sum_{j=1}^{J} \alpha_j Sign(\mathbf{w}_j^T\mathbf{x} + w_0)\right)$$

Motivations



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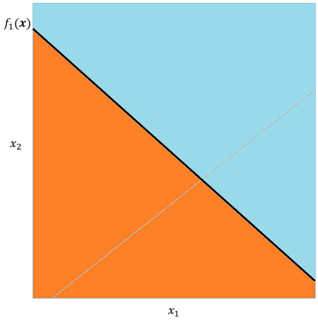

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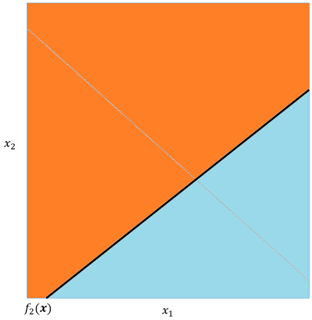

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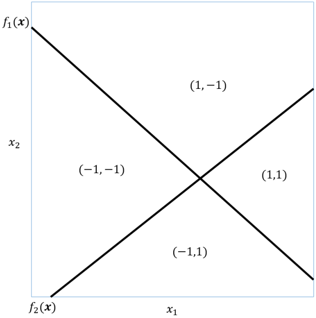
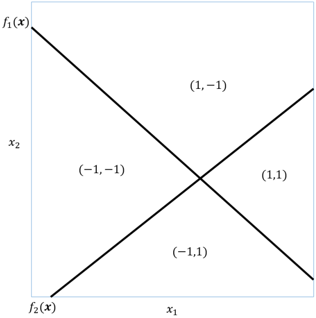


Motivations

- 'AND' operator: $AND(f_1, f_2)$



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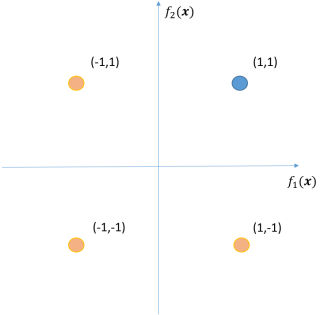

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- 'AND' operator: $AND(f_1, f_2)$
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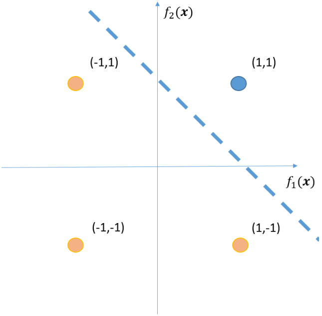
$$\alpha_0 + \alpha_1 f_1 + \alpha_2 f_2 = 0$$



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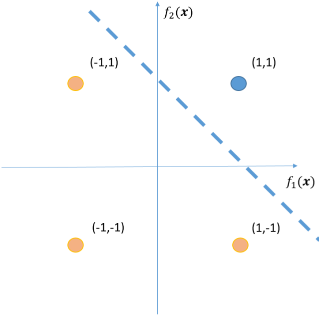

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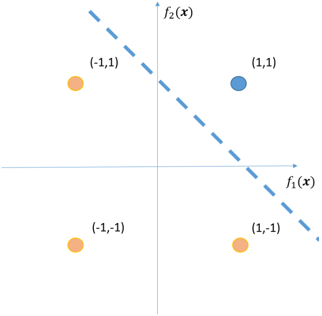

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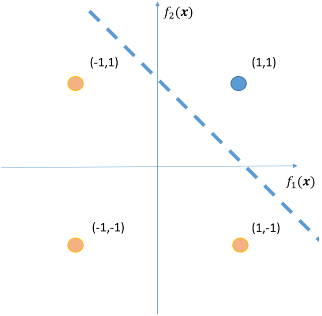

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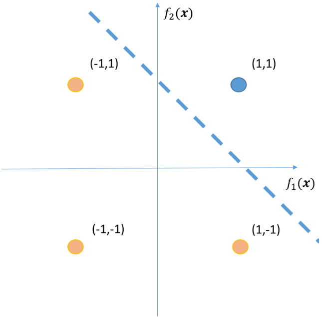

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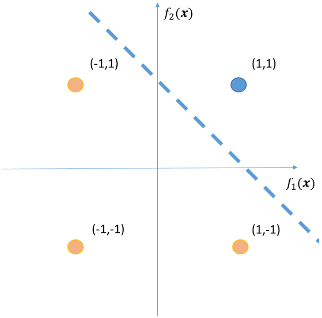

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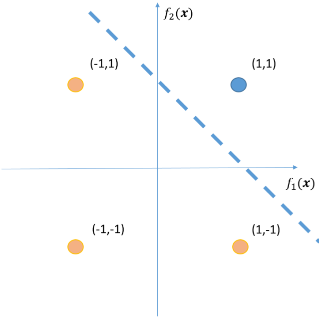

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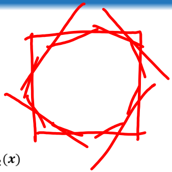
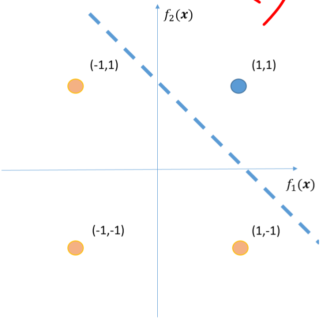
- 'OR' and 'NOT'
- Any convex set can be approximated by this model if J is large enough.





Non-uniform Aggregation of Perceptrons

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$$F(\mathbf{x}) = Sign\left(w_0^{(2)} + \sum_{j=1}^{J} w_j^{(2)} Sign\left(\sum_{i=1}^{d} w_{j,i}^{(1)} x_i + w_{j,0}^{(1)}\right)\right)$$

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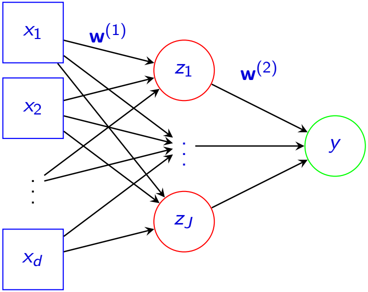

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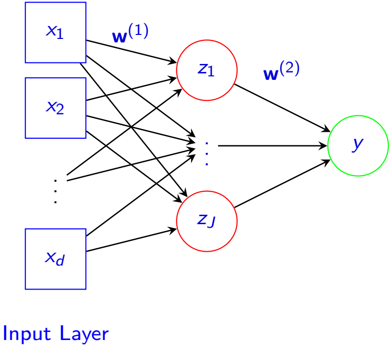

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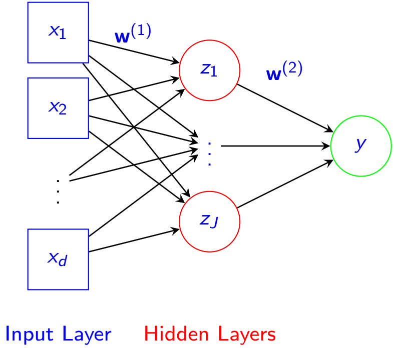

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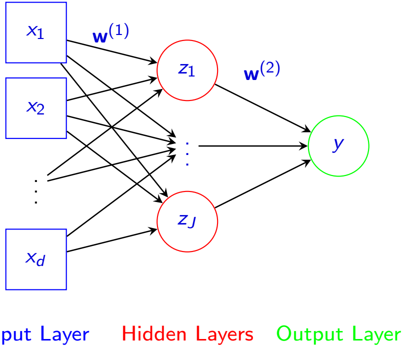
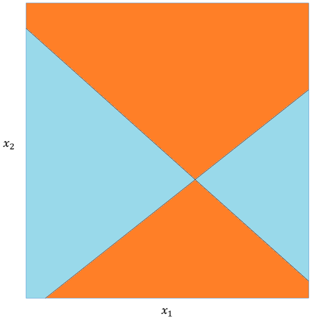


Motivations

- A non-convex set.



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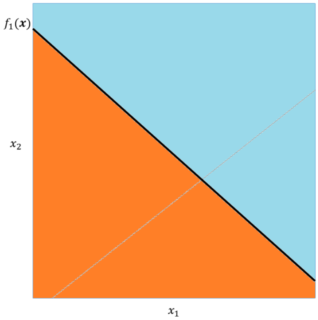

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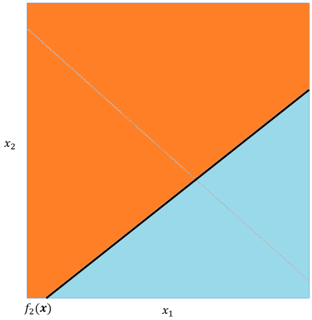

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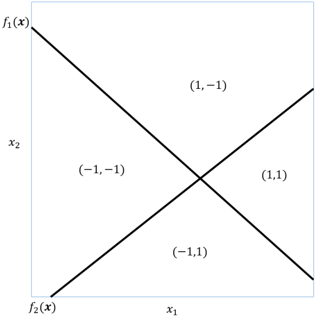

Motivations

- A non-convex set.
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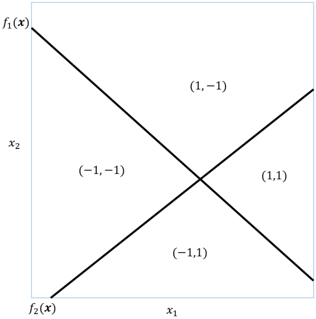

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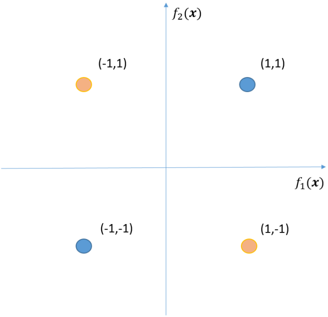

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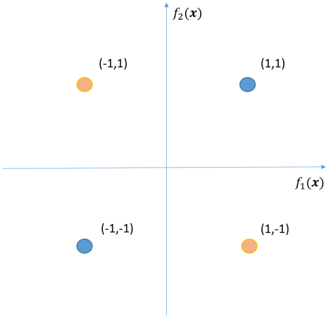

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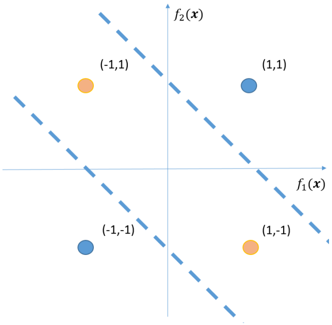

Motivations

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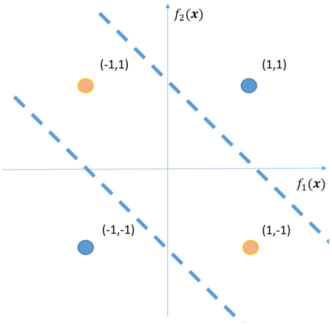

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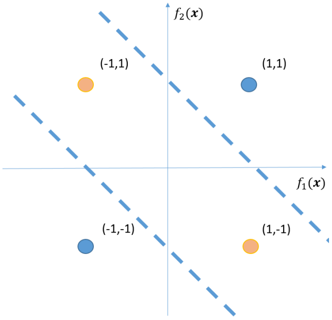
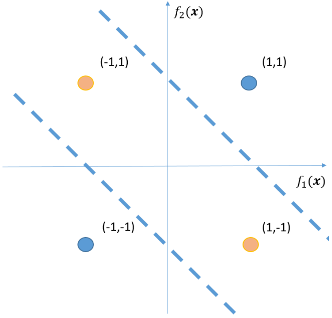


There is nothing difficulty cann't be solved by eating a hamburg at MÄX, if so, just eat one more.

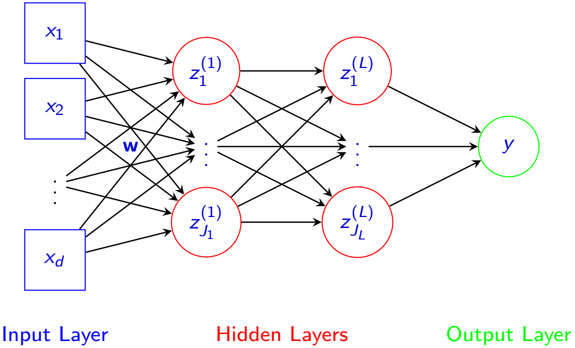
Motivations

- A non-convex set.
- 'XOR' operator.
- Not linear separable... Any idear?
- Multi-Layers Perceptron (MLP).



There is nothing difficulty cann't be solved by Multi-layers perceptron, if so, just add one more layer

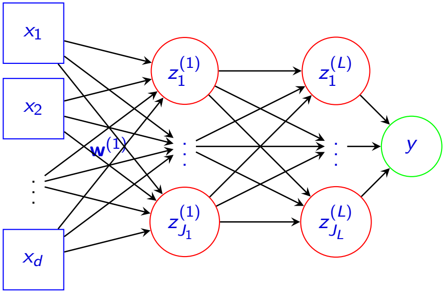
Multi-Layers Perceptron



From MLP to Artificial Neural Network

- The general MLP can be represented as

$$
\begin{aligned}
z_j^{(1)} &= Sign\left(\mathbf{w}_j^{(1)T}\mathbf{x}\right)\\
&\vdots\\
z_j^{(L)} &= Sign\left(\mathbf{w}_j^{(L)T}\mathbf{z}^{(L-1)}\right)\\
y &= Sign\left(\mathbf{w}_j^{(L+1)T}\mathbf{z}^{(L)}\right)
\end{aligned}
$$



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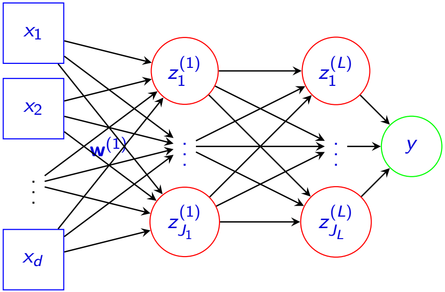
- Neurons: $z_j^{(L)}$, latent variable.



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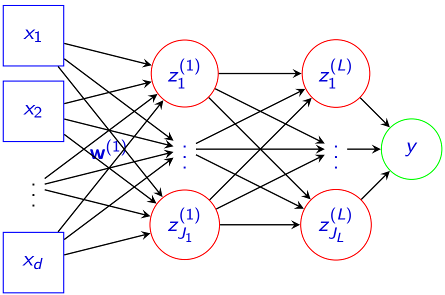
- Activation function, $\sigma(\cdot)$: $Sign(\cdot)$, Identity function, logit function and so on.

z : S



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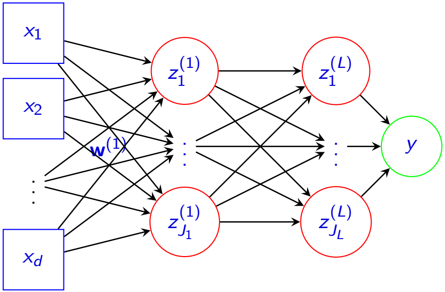

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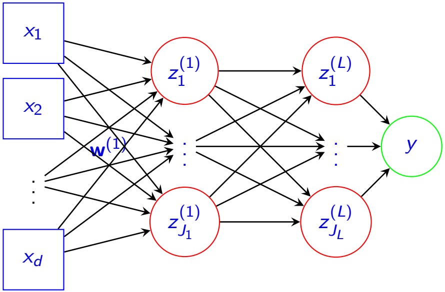

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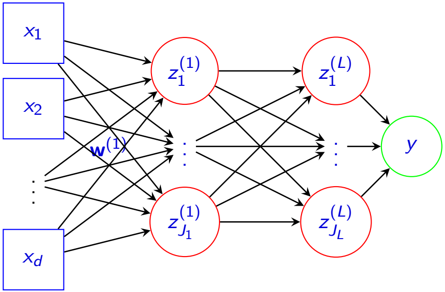
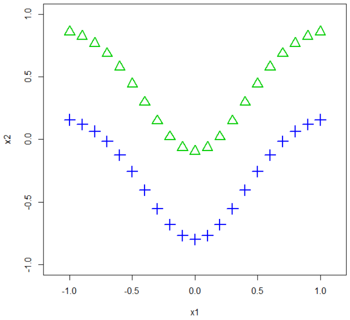
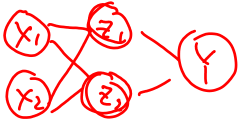


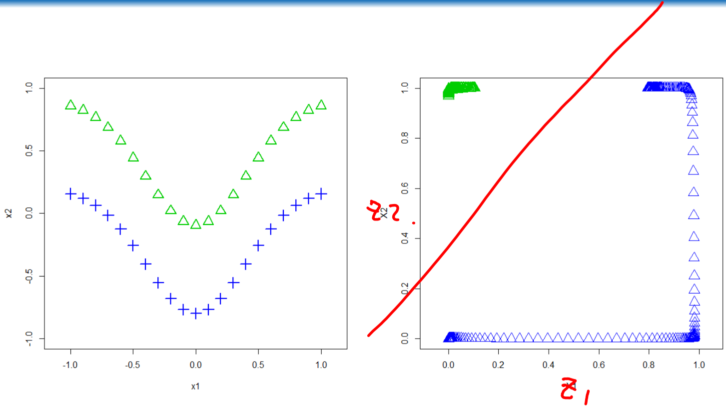
Feed-foward Neural Network

ANN, an integrated learning process

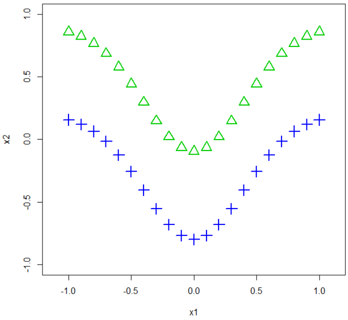


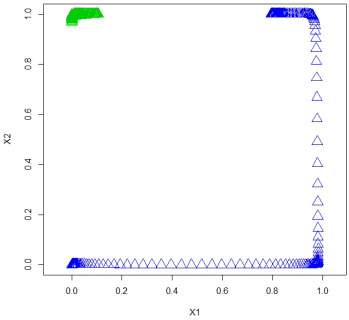


ANN, an integrated learning process



ANN, an integrated learning process





- The last layer of neurons can be viewed as a set of extracted features from the raw data.
- ANN can be viewed as an integrated learning process.