Phase Space Modulations in Magnetised Plasmas By a Mildly Relativistic Two-Stream Instability

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Abstract. By using a kinetic particle-in-cell simulation and point-rendering visualisation techniques, we investigate a two-stream plasma instability possibly found in the accretion disc of black holes. We consider a plasma in an oblique magnetic field. The instability gives rise to an electrostatic wave able to trap electrons and accelerate them by cross-field transport. Our results show a peak acceleration of the electrons to speeds remarkably similar to the bulk speed of microquasar jets.

1 Introduction

Ionised gas, or *plasma*, is by far the most common state for all known matter in the universe. Generally, a space plasma may be considered quasi-neutral and collisionless. Such plasmas are intrinsically non-linear and support a multitude of instabilities.

One of the most spectacular and puzzling astrophysical plasma processes known is that of highly relativistic *jets* ejected close to the magnetic poles of accreting black holes in, e.g., active galaxies and microquasars. Observational evidence[1][2] indicates that the accretion disc of inflowing plasma is instrumental in the jet development, but the underlying mechanisms are still unresolved. What is clear, though, is the existence of a very powerful acceleration scheme. We investigate a particle acceleration mechanism similar to *electron surfing acceleration*[3][4][5] An electrostatic wave (ESW), propagating across an external magnetic field, saturates by trapping electrons that are accelerated due to the cross-field transport.

In line with previous works[3][4][5][6], we consider an instability consisting of two counter-propagating proton beams in an electron-proton plasma. Proton beams are strong non-thermal occurences in accretion discs and are, for example, produced by shocks[4][7]. An ESW grows with a phase speed approximately equal to the beam speed. In the present work, we treated mildly relativistic ESWs[5][6] and introduced an oblique magnetic field to induce a modified electron surfing acceleration (MESA). This has been done for highly relativistic phase speeds[3][4], where MESA was suggested a potential accelerator in the context of active galactic nuclei and gamma ray bursts. In this paper, we choose conditions that are more likely for the less energetic microquasar.

We use a kinetic particle-in-cell (PIC) simulation that solves the Maxwell equations and the relativistic Lorentz equation of motion[8]. To analyse multi-dimensional simulation data, we have implemented a visualisation tool based on a point-rendering technique. Interactive 3D graphics with animation features readily exposes the time evolution of wave induced phase space structures.

The electrons form characteristic trapped particle vortices, indicating magnetic field acceleration but we also find smaller vortices that are coupled to the electrostatic wave acceleration. The electron-wave interaction gives a peak Lorentz factor for the electrons corresponding to what is probable in the bulk flow of microquasar jets. Using dilute proton beams we observe strong non-linear effects in the proton population that maintain large electrostatic fields after the initial two-stream driver wave has collapsed.

2 Kinetic Particle-In-Cell Simulation and Visualisation

Kinetic theory dictates plasma a phase space fluid defined by the position and velocity vectors. The numerical code represents the fluid as an ensemble of computational particles. For a plane ESW we need only to resolve one spatial dimension, called x. The velocity dimensions are all resolved due to the oblique magnetic field. The cross velocity accelerates trapped electrons perpendicular to the magnetic field and couples it to the acceleration by the ESW. We use periodic boundary conditions such that any particle traveling out of the box is re-inserted. The proton beams counter-propagate in the simulation box with the mean speed $v_b = 0.6c$, with c the speed of light, giving an initial zero net current.

A plasma is defined by a set of characteristic parameters where the electron plasma frequency $\omega_{p,e} = \left(n_e e^2/m_e\epsilon_0\right)^{1/2}$ is one; n_e, e, m_e and ϵ_0 are the electron particle density, the elementary charge magnitude, the electron rest mass and the dielectric constant respectively, and the Debye radius, $r_D = v_{th,e}/\omega_{p,e}$, another. Here $v_{th,e} = \left(k_B\hat{T}_e/m_e\right)^{1/2}$ is the thermal electron velocity, where k_B is Boltzmann's constant and $\hat{T}_e$ is the electron temperature.

The simulation enforces $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$, ρ is charge density, and $\nabla \cdot \mathbf{B} = 0$ and solves the Maxwell-Lorentz equations

$$\nabla' \times \mathbf{E}' = -\frac{\partial \mathbf{B}'}{\partial t'}, \tag{1}$$

$$\nabla' \times \mathbf{B}' = \mathbf{j}' + \frac{\partial \mathbf{E}'}{\partial t'}, \tag{2}$$

$$\frac{d\left(\mathbf{v'}_j \Gamma(\mathbf{v'}_j)\right)}{dt'} = \frac{q'_j}{m'_j}\left(\mathbf{E}' + \mathbf{v}' \times \mathbf{B}'\right). \tag{3}$$

Physical variables are normalised to make our results more generally applicable, as described in Ref.[4]. Thus, the electric and magnetic fields are written $\mathbf{E} = cm_e\omega_{p,e}\mathbf{E}'/e$, $\mathbf{B} = m_e\omega_{p,e}\mathbf{B}'/e$ and the current density $\mathbf{j} = n_e ce\mathbf{j}'$, unprimed quantities have physical units. In the same manner, we substitute time and space variables as $t = t'/\omega_{p,e}$ and $x = cx'/\omega_{p,e}$. The velocity is expressed as $\mathbf{v} = c\mathbf{v}'$ and the charge and mass of particle j are $q_j = eq'_j$ and $m_j = m_e m'_j$. The gamma factor is denoted by $\Gamma(\mathbf{v}'_j) = (1 - v'^2_j)^{-1/2}$, where v'_j is the normalised speed of particle j.

The initial thermal velocities for all of the four plasma species are much less than the beam speed and we use a cold plasma approximation[9]. The most unstable wave solution to this approximation has the wavenumber $k_u \approx \omega_{p,e}/v_b$ and the corresponding frequency $\omega_u \approx \omega_{p,e}$. The simulation box length, $L = 8\lambda_u = 16\pi/k_u$, is resolved by 3200 cells with cell-length $\Delta_x \approx 0.7r_D$. Each particle species is represented by $N = 115200$ computational particles and we follow the instability over a total time period of $T_t = 500T_p$, with $T_p = 2\pi/\omega_{p,e}$. The magnetic field is $\mathbf{B} = B_0(1/\sqrt{2}, 0, 1/\sqrt{2})$ and the ratio $\omega_c = eB_0/m_e \approx 7.1\omega_{p,e}$.

Interactive Point-Rendering We have based our visualisation application on C++ and OpenGL, which makes it high performing and very flexible. Using point-rendering and the interaction possibilities in 3D computer graphics we map the simulation data onto the window screen and manipulate it there. The rendering is straightforward where each particle phase space coordinates, (x, v_x, v_y), correspond to a 3D window position. The graphical object is a pixel-sized point and the fourth coordinate, v_z, is displayed as the point colour.

The data set can be translated in the window Z-plane or rotated about the X- and Y-axes. It is also possible to zoom in and out of the data set as well as scale each axis separately. The latter is useful when examining small non-symmetrical scale properties, for instance. A time animation readily discloses the dynamical properties of a plasma, as demonstrated in Refs.[3][4][5][10].

3 Result and Conclusions

Initially the electrostatic field grows exponentially until it saturates by trapping electrons in the wave potential. The visualisation reveals a large spatio-temporal range and we see the electron dynamics on two time scales; fast small vortices form in the trapping process and large vortices, on a slower time scale expanding in v_y, are due to the acceleration across the magnetic field direction. Thin proton beams are susceptible to energy depletion effects and the modulation of the beam population maintains strong electrostatic fields after the collapse of the initial ESW. Figure 1 shows a snapshot where vortices of trapped electrons are beginning to form. Shortly afterwards the electrons are detrapped by the collapse of the ESW. At that time they have reached a peak Lorentz factor along the magnetic field of about four. This would allow them to overcome the gravitational pull from the black hole and escape into outer space with speeds close to what is observed in microquasar jets.

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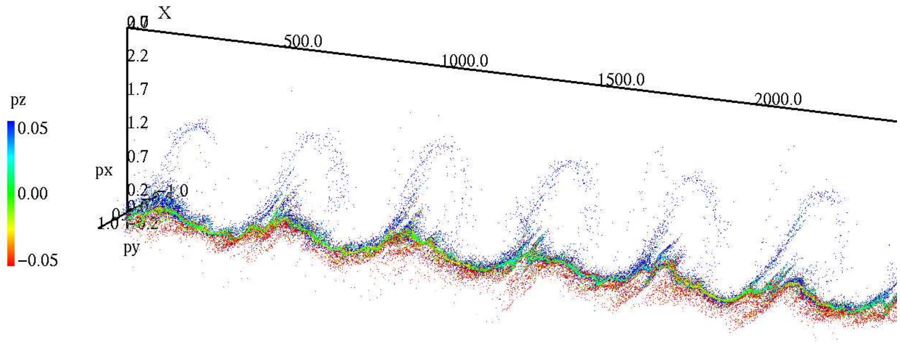

Fig. 1. Trapped particle islands. Note that we display the normalised momentum phase space, i.e. $\mathbf{p} \equiv \Gamma(v_b)\mathbf{v}/c$, where $\Gamma(v_b) = (1 - v_b^2/c^2)^{-1/2}$ is the gamma factor. X is the spatial coordinate as in the simulation box. Colour values are clamped to the maximum and minimum as shown in the left bar.

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