Hybrid Simulation of the Auroral Current Circuit

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Abstract. The aurora is created by electrons that are accelerated along the Earth's magnetic field lines before they impinge on the ionosphere and create light through excitation processes. To model the large-scale processes involved in the generation of auroras it is common to describe the electrons as a fluid, where the fluid equations are solved self-consistently together with Maxwell's equations. However, as the electrons are accelerated they are also heated, and no simple equation of state can be utilized when closing the hierarchy of fluid equations. Therefore, we present a hybrid model where the fluid equations are complemented by a particle simulation. By letting the electric field from the fluid simulation accelerate the particles, and then using the temperature of the particle distribution as a feed-back to the fluid equations, we obtain a self-consistent description of the auroral electrons.

1 The Auroral Current Circuit

In the tail of the Earth's magnetosphere there is a generator region that contains processes driving an ion current perpendicular to the magnetic field lines. At the flanks of the generator region the perpendicular current is diverted into field-aligned currents connecting the generator to the ionosphere. In the ionosphere these field-aligned currents are closed by a perpendicular current. Thus, we have a current circuit, the auroral current circuit, where the upward field-aligned current is carried by downgoing electrons that create auroras. The geometry of the auroral current circuit and the generator region in the equatorial magnetosphere is sketched in Figure 1.

2 Model

2.1 Fluid Simulation

The electron fluid is in our model described by the equations

$$\partial_{\mathsf{t}}\mathsf{E_x} = -\mathsf{A}^2\partial_{\mathsf{z}}\mathsf{B_y} - (1-\mathsf{A}^2)\mathsf{F}\ , \tag{1a}$$

$$\partial_{\mathsf{t}}\mathsf{E_z} = \frac{B_z}{B_0}\left(\partial_{\mathsf{x}}\mathsf{B_y} + \mathsf{j_z}\right)\ , \tag{1b}$$

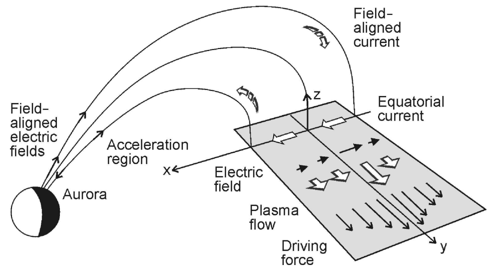

Fig. 1. The geometry of the auroral current circuit and the generator region in the equatorial magnetosphere. The curvature of the magnetic field lines is neglected in our model equations.

$$\partial_{\mathsf{t}} \mathsf{B}_{\mathsf{y}} = \partial_{\mathsf{x}} \mathsf{E}_{\mathsf{z}} - \partial_{\mathsf{z}} \mathsf{E}_{\mathsf{x}} \, , \tag{1c}$$

$$\partial_{\mathsf{t}} \mathsf{n} \; = -\partial_{\mathsf{z}} \mathsf{j}_{\mathsf{z}} \, , \tag{1d}$$

$$\partial_{\mathsf{t}} \, \mathsf{j}_{\mathsf{z}} = -\frac{m_i}{m} \mathsf{n} \mathsf{E}_{\mathsf{z}} - \partial_{\mathsf{z}} \left(\mathsf{n} \mathsf{T}_{\mathsf{z}} + \frac{\mathsf{j}_{\mathsf{z}}^2}{\mathsf{n}} \right) - \mathsf{n} \mathsf{T}_{\perp} \frac{\partial_{\mathsf{z}} B_z}{B_z} \, , \tag{1e}$$

which are the non-zero components of Ampere's and Faraday's laws together with the equation of continuity and the momentum equation. Here we use simulation variables, where E_{x} and E_{z} are the electric fields, B_{y} is the perpendicular magnetic field, A is the Alfvén velocity, F is the generator force, n is the electron density, j_{z} is the field-aligned electron current, while T_{z} and $\mathsf{T}_{\perp}$ are the field-aligned and perpendicular temperatures. $B_z(z)$ is the geomagnetic field and B_0 is the field strength at $z = 0$ in the equatorial plane. The mass ratio in the last equation is the ion mass m_i divided by the electron mass m.

This type of model for auroras have been considered throughout the last decades, e.g. [1],[2] and [3]. However, those models have assumed the electrons to be cold or isothermal, and they have not been able to reproduce the large field-aligned electric fields seen in satellite observations. From the recent study [4] we know that temperature variations play a crucial role in supporting these electric fields. To determine the temperatures we need some equation of state closing the hierarchy of fluid equations. However, to find such an equation of state applicable to auroral electrons is, as discussed in [4], a difficult task as the temperatures at one position along the field line depend on the history of the entire auroral flux tube.

2.2 Particle Simulation

To close the set of fluid equations, we use a particle simulation that runs parallel to the fluid simulation. We accelerate the particles by the electric field from the fluid solver, which implies that the particle motion will be consistent with the fluid simulation. From the velocity distribution of the electrons we can determine temperatures that are consistent with the fluid simulation and can be used in the momentum equation.

The particles move along z with velocity v_z and gyrate the field line with velocity $v_\perp$. Their position and velocity are updated according to

$$d_t z = v_z \tag{2}$$

$$d_t v_z = \frac{-e}{m} E_p - \mu \partial_z B_z \tag{3}$$

where $-e$ is the electron charge, $\mu = m v_\perp^2 / 2B_z$ is the conserved magnetic moment of the particle, and E_p is the electric field that accelerates the particle. The electric field E_p is equal to the parallel electric field in the fluid simulation, but with two corrections to ensure that the density and current of the particles are equal to the density and current of the fluid.

The main idea of the hybrid simulation is similar to a regular Particle-In-Cell (PIC) code with a particle mover and a field solver. There are, however, two main differences. First, the system of Maxwell's equations in the field solver has been extended by the two lowest order fluid equations. Second, the feedback from the particles to the field solver is done through the temperatures instead of the current as in an electromagnetic PIC code or the density as in an electrostatic PIC code.

3 Implementation

The fluid equations are solved using an implicit algorithm described in detail in [3], on a grid of size $n_x \times n_z = 27 \times 100$. By rewriting the equation system as two block-tridiagonal set of equations, they can be solved in $\mathcal{O}(n_x \times n_z)$ operations.

Since the particles are guided by the magnetic field lines and therefore only move along z, the total particle simulation can be considered as n_x independent sub-simulations. For each such sub-simulation the particles are initially loaded to be consistent with a certain density $n_0(z)$ and temperature $T_0(z)$ in the fluid simulation. During the simulation, particles are continuously removed and injected at the boundaries at the generator and the ionosphere. The injection rate is determined from the density and temperature at the boundaries. To handle this adding and removing of particles in a sufficiently fast manner the particles are implemented as a linked list. In each sub-simulation we need $\sim 10^7$ particles to obtain reasonably good statistics when computing the temperatures from the particle distribution. The particle simulation is therefore demanding with respect to the amount of memory and computer time needed.

When considering the computer time used in the hybrid model, the time to solve the fluid equations is negligible compared to the time to accelerate, move, and compute the velocity moments from the particles. Since almost all time is spent in the particle simulation and the particle simulation can be considered as nx independent sub-simulations, the hybrid model can easily be parallelized on nx processors with an efficiency of nearly 100% due to the minimal amount of inter-processor communication.

4 Results

This model gives the first description of the auroral build-up in the presence of a self-consistent electron temperature variation, and the results are, in contrast to earlier models (e.g. [1],[2] and [3]), consistent with observations and they converge to a quasi-stationary state consistent with stationary kinetic theory. Furthermore, the hybrid model describes interesting, previously unrevealed transient effects during the auroral build-up.

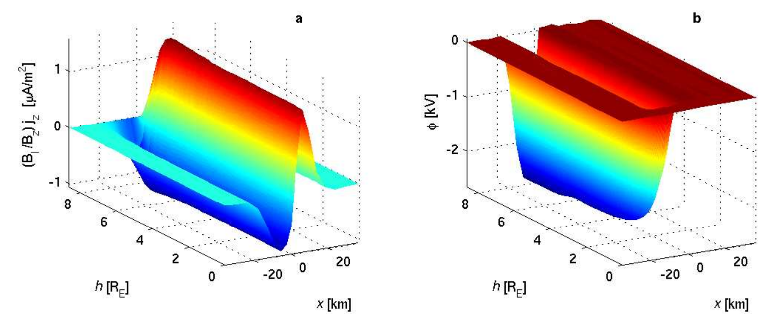

Fig. 2. Plots showing the quasi-stationary state of (a) the field-aligned current density mapped to the ionosphere, and (b) the potential $\phi = -\int E_z dz$, as functions of x and altitude h given in Earth radii $\mathrm{R_E}$. The snapshots show the upward current (negative) and the downward current (positive), and the large field-aligned potential drop in the upward current region.

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