Numerical performance of a parallel solution method for a heterogeneous 3D Helmholtz equation

A. Kononov[1], C. Riyanti[2], C. Oosterlee[2], and C. Vuik[2]

[1] Computational Physics Group, TU Delft, 2628 BL Delft, The Netherlands
a.v.kononov@tnw.tudelft.nl,
[2] Delft Institute of Applied Mathematics,
TU Delft, 2628 CD Delft, The Netherlands

Abstract. The parallel performance of a numerical solution method for the scalar and vector 3D Helmholtz equation written for inhomogeneous media is studied. The numerical solution is obtained by an iterative method applied to the preconditioned linear system which has been derived from either finite difference or finite element discretization. Parallel execution is implemented using both the MPI library and Open MP. The analysis focuses on the most difficult cases which may cause considerable difficulties for Helmholtz type problems such as high wave-numbers and highly irregular media parameters.

The Helmholtz equation, which is also called a reduced wave equation, in scalar or vector form is often used to approximately model wave propagation in inhomogeneous media. The demand for reliable numerical solutions to such type of problems is repeatedly encountered in geophysical and optical applications. In geophysical applications, for example, wave propagation simulations are used for the development of acoustic imaging techniques for gaining knowledge about geophysical structures deep within the Earth's subsurface.

The discretization of the corresponding Helmholtz problem is usually based on finite difference (FD) or finite element discretization (FEM) schemes which are relatively simple and, at the same time, effective and increasingly popular. However, in order to maintain acceptable numerical accuracy in the FD or FEM solutions, fine enough grid spacings per wave length need to be employed [1]. This implies that for most realistic cases the penalty in terms of computational costs and memory requirements is tending to be extremely high.

These severe limitations may effectively be resolved by using the power of multiprocessor computer architectures, such as, for example, Linux computer clusters. Unlike direct solution methods, iterative methods allow effective parallelization and require less memory utilization, and thus enable one to compute the solution of Helmholtz problems of practical size in reasonable time.

In the first part of the present paper, we study the parallel performance of the iterative solver for the scalar 3D Helmholtz discretized by finite difference scheme. We focus on real-life applications e.g., in seismic wave propagation and inversion problems in geophysics [3]. The parallel approach, which is proposed in the present paper is rooted from the sequential code that is based on the method described in [2]. In particular, we employ a complex shifted-Laplace operator as preconditioner inside the Bi-CGSTAB iterative method and use one multigrid iteration to approximately compute the inverse of the resulting preconditioner operator. This method appears to be advantageous because it is relatively simple and effective for the problems in which the media parameters can be strongly heterogeneous and the acoustic frequencies and the corresponding wavenumbers are relatively high (geophysical applications). The parallel program is implemented using both the standard MPI library and Open MP. The parallel scalability of the program is very satisfactory. This is mostly due to the well parallelizable multigrid preconditioner that consumes most of the computational time.

The second part of the paper is devoted to the parallel solution of the vector 3D Helmholtz problem. The corresponding linear system is obtained using FEM discretization of the time-harmonic 3D Maxwell's equations applied to the optical problems. The numerical solution is obtained by an iterative method, namely the parallel Bi-CGSTAB, applied to the preconditioned linear system. The preconditioner is factorized explicitly using the parallel ILU-procedure. At the current stage, using Open MP the parallelization of the sequential code has been accomplished targeting the shared memory architectures: dual-processor or quad-processor nodes (meanwhile, extension to the distributed memory architectures is under development). The approach used shows a quite satisfactory parallel performance which is mostly due a proper reordering strategy applied to the matrix prior the factorization phase.

References

1. Marfurt, K.J.: Accuracy of finite-difference and finite element modeling of the scalar and elastic wave equations.Geophysics. **49:5** (1984) 533–549
2. Erlangga, Y.A., Oosterlee, C.W., Vuik, C.: A novel multigrid based preconditioner for heterogeneous Helmholtz problems. SIAM J. Sci. Comput. **27** (2006) 1471–1492
3. Bourgeois A., Bourget M., Lailly P., Poulet M., Ricarte P., Versteeg R.: Marmousi, model and data, in Versteeg, R., and Grau, G., Eds. The Marmousi experience. Proceedings of the 1990 EAEG workshop on Practical Aspects of Seismic Data Inversion: Eur. Assoc. Expl. Geophys. (1991) 5–16